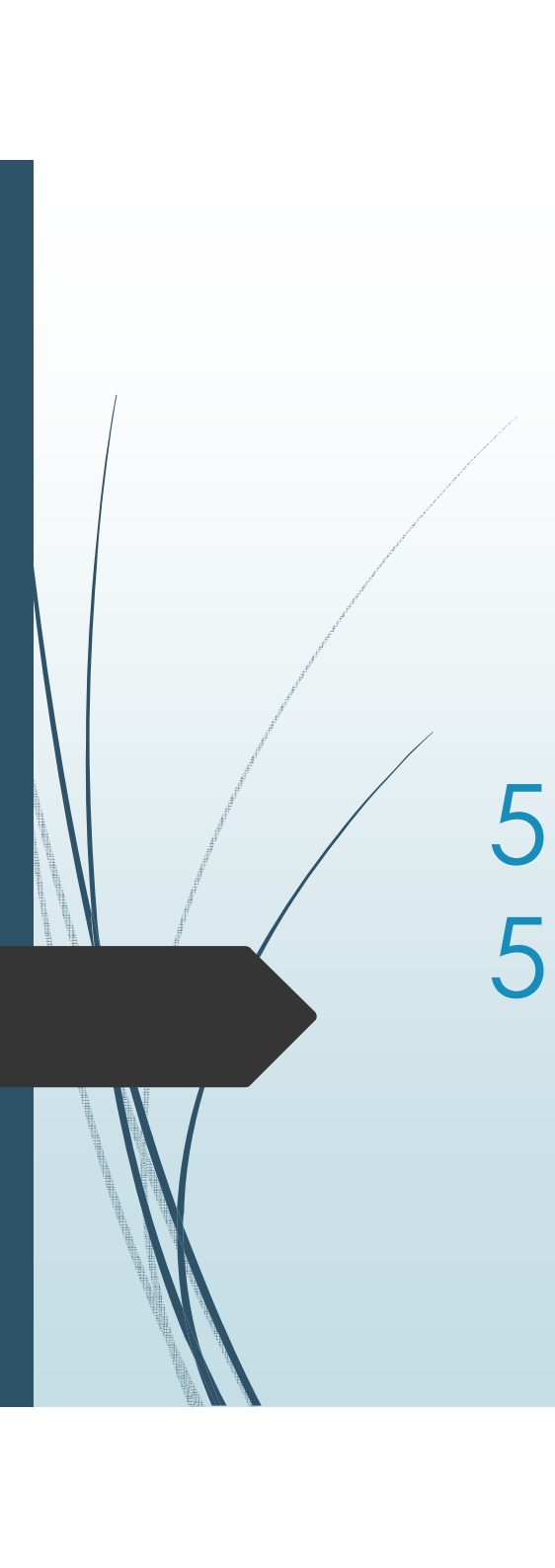




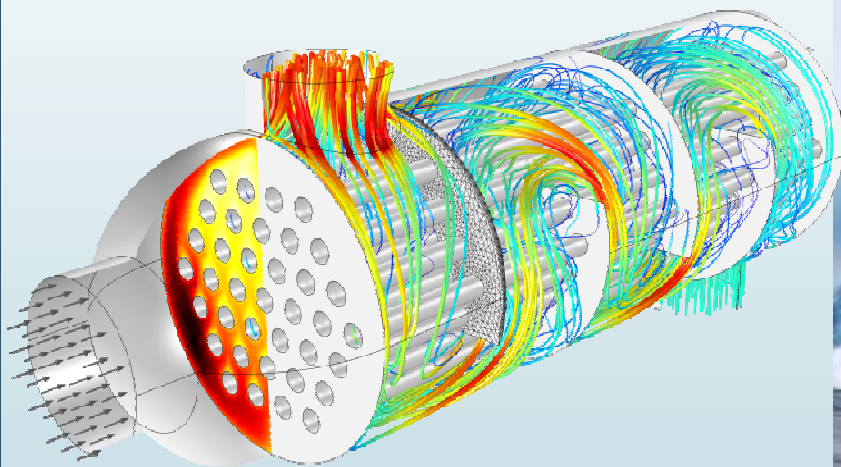
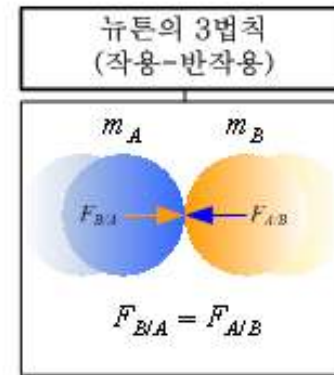
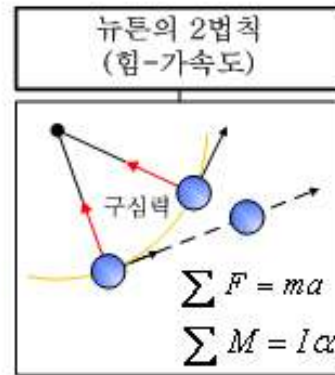
5.3 평행평판 주위의 난류경계층 5.4 수직평판, 박리 및 압력항력

조선해양공학과 산업대학원

201674184

김일도

Newton의 Principia (1687)



유체력에 관한 세 개념

- 경계층 - 마찰항력(frictional drag)
- 박리(separation) - 압력항력(pressure drag)
- 순환(circulation) - 양력(lift)
- 만약 움직이는 유체에 점성이 없다면?

-> 유체는 비점성, 비회전성

-> Euler 운동 방정식 (Streamline)(1755)

$$\frac{\partial u_s}{\partial t} + \frac{\partial}{\partial s} \left(\frac{1}{2} u^2 \right) + \frac{1}{\rho} \frac{\partial p}{\partial s} + g \frac{\partial z}{\partial s} = 0$$

유체입자의 가속도

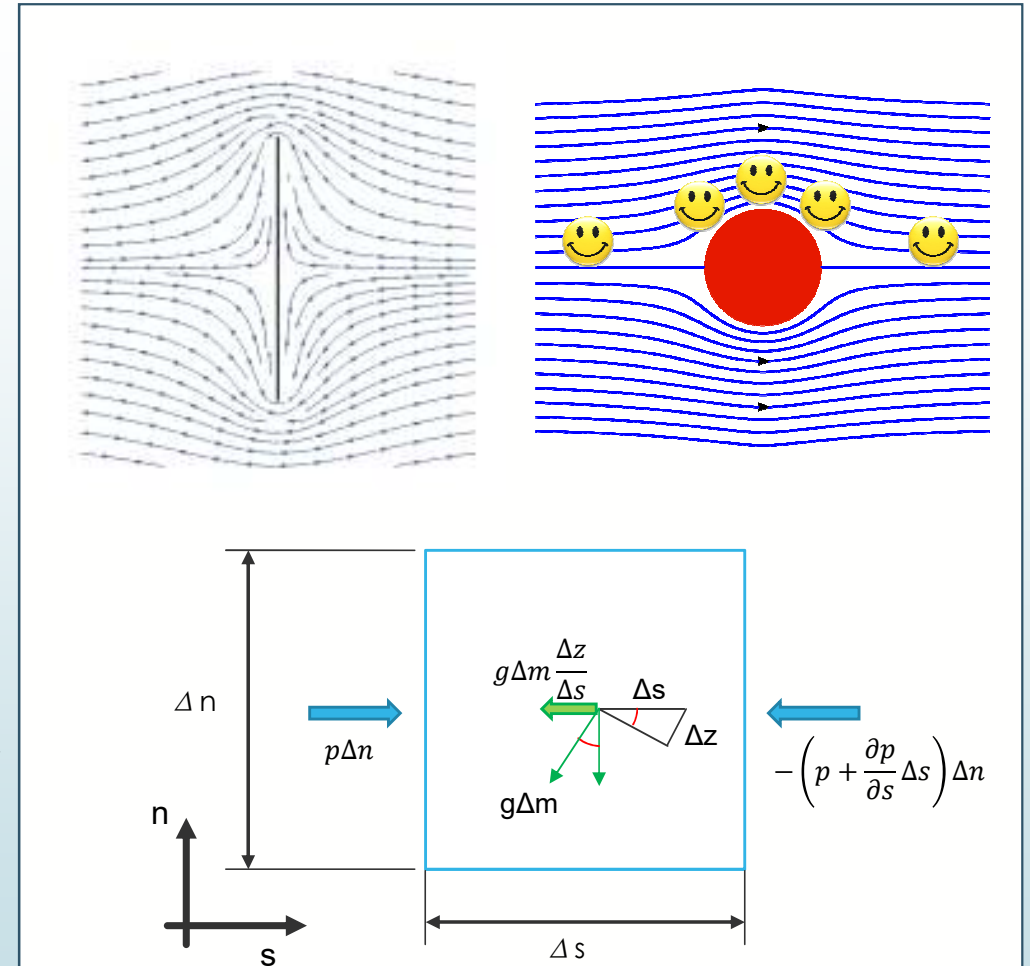
압력

중력

-> Bernoulli 방정식 (Streamline, 정상유동)(1738)

$$\frac{1}{2} u^2 + \frac{p}{\rho} + gz = \text{constant}$$

-> 뉴턴의 운동법칙에 따라 물체에 작용하는 유체력은 영이다.



Cauchy 운동방정식과 Navier-Stokes Equation

- 점성 유체 내부에 작용하는 내력의 일반적인 상태를 표현(1822)
- Traction과 응력 Tensor 개념을 도입하여 운동방정식을 유도
- 단위 면적 당 내력 = Traction (작용면에 평행(전단응력), 수직인 성분(압력))

$$\begin{array}{l}
 \text{X-direction} \quad -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} \right) + g_x = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \\
 \text{y-direction} \quad -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{1}{\rho} \left(\frac{\partial \tau_{yy}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \right) + g_y = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}
 \end{array}$$

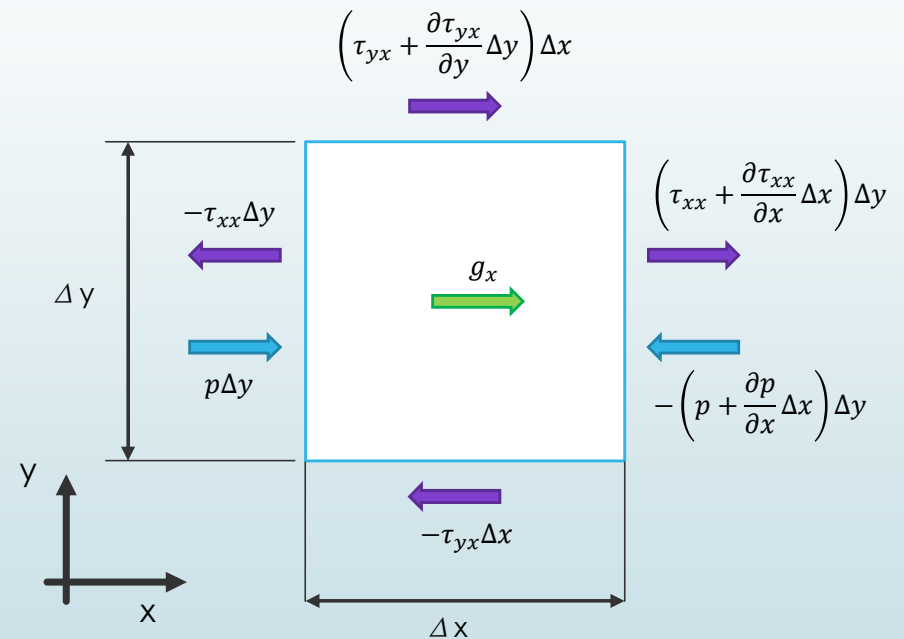
압력 점성력 중력 유체입자의 가속도

$$\frac{1}{\rho} \nabla \cdot \sigma + g = \frac{Du}{Dt}$$

- Navier-Stokes Equation(1840)

$$\tau_{xx} = 2\mu \frac{\partial u}{\partial x}, \tau_{yx} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

$$\begin{array}{l}
 \text{X-direction} \quad -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \rho g_x = \rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) \\
 \text{y-direction} \quad -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \rho g_y = \rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right)
 \end{array}$$



평행평판 주위의 층류경계층

- 압력 = 0, y 방향 유동 ≈ 0

- Prandtl Boundary Layer (1904)

Mass (질량보존) $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

Momentum (운동방정식) $-\cancel{\rho} \frac{\partial \cancel{v}}{\partial x} + \mu \left(\cancel{\frac{\partial^2 \cancel{v}}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} \right) + \cancel{\rho} g_x = \rho \left(\cancel{\frac{\partial v}{\partial t}} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right)$

- 우리가 알고 싶은 사항은? 평판에서의 유체력 -> 전단응력 -> 속도의 변화량 -> 계속 변경 -> 속도의 x 방향 변화량을 나타낼 수 있는 무차원 함수를 풀자???

- Blasius Solution (1908) (PDE -> ODE)

Governing Equation

Conservation of Mass

Momentum Equation

Boundary Condition: $u(y=0)=0$, $u(y=\delta)=U$

Stream function, $\psi(u = \partial\psi/\partial y, v = -\partial\psi/\partial x)$ introduced ???

$$\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = \nu \frac{\partial^3 \psi}{\partial y^3}$$

Similarity solution

$$\frac{u}{U} = g(\eta) = \frac{df}{d\eta}, \quad \eta \propto \frac{y}{\delta(x)}$$

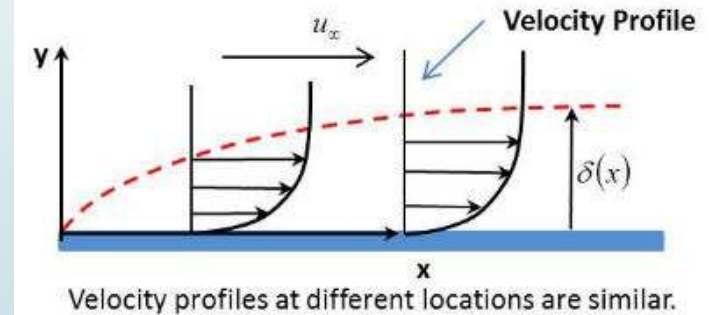
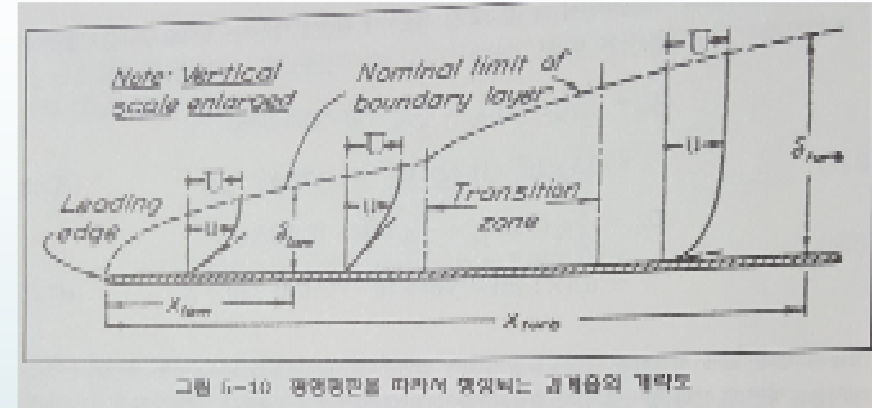
$$\psi = U \delta f(\eta)$$

$$f''' + \left(\frac{U \delta}{\nu} \frac{d\delta}{dx} \right) f f'' = 0$$

$$= \text{Constant}$$

$$\delta = \sqrt{\frac{\nu x}{U}}$$

$$\eta = y \sqrt{\frac{U}{\nu x}}$$



평행평판 주위의 층류경계층

Boundary layer thickness (at $u/U = 0.99$)

$$\frac{\delta}{x} = \frac{5}{\sqrt{Re}}$$

Shear stress at wall is

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \eta} \frac{\partial \eta}{\partial y} = 5 \frac{U}{\delta} \frac{d}{d\eta} \left(\frac{u}{U} \right)$$

$$\tau_0 = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0} = \mu C \frac{U}{\delta}$$

Friction coefficient

$$c_f = \frac{\tau_0}{\rho U^2 / 2} = \frac{0.664}{\sqrt{Re}}$$

Drag force for friction

$$D_f = b \int_0^l \tau_0 dx$$

Drag coefficient for friction

$$C_{D,f} = \frac{D_f}{b l \rho U^2 / 2} = \frac{1.328}{\sqrt{Re}}$$

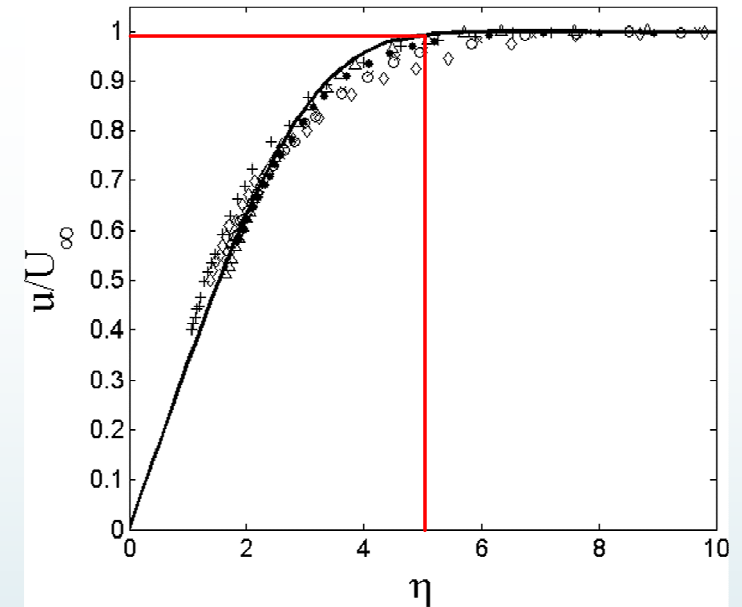
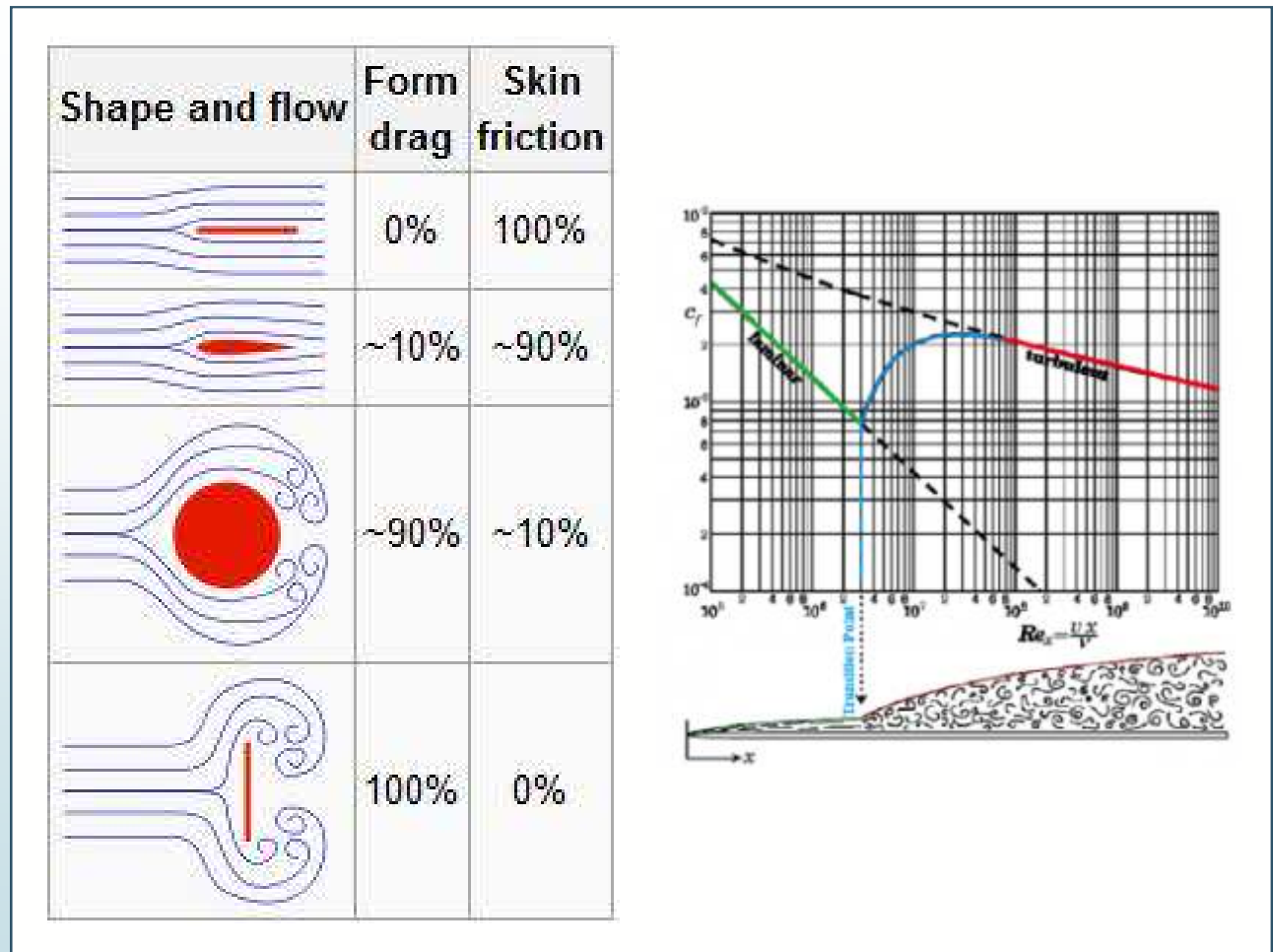


TABLE 14.1 Blasius Solution

η	f	$\frac{\partial f}{\partial \eta}$	$\frac{\partial^2 f}{\partial \eta^2}$
0	0	0	0.3321
0.5	0.0415	0.1659	0.3309
1.0	0.1656	0.3298	0.3230
1.5	0.3701	0.4868	0.3026
2.0	0.6500	0.6298	0.2668
2.5	0.9963	0.7513	0.2174
3.0	1.3968	0.8460	0.1614
3.5	1.8377	0.9130	0.1078
4.0	2.3057	0.9555	0.0642
4.5	2.7901	0.9795	0.0340
5.0	3.2833	0.9915	0.0159
5.5	3.7806	0.9969	0.0066
6.0	4.2796	0.9990	0.0024
6.5	4.7793	0.9997	0.0008
7.0	5.2792	0.9999	0.0002
7.5	5.7792	1.0000	0.0001
8.0	6.2792	1.0000	0.0000

- Laminar boundary layer,
 $Re_c = 2 \times 10^5$



난류경계층 형성 과정

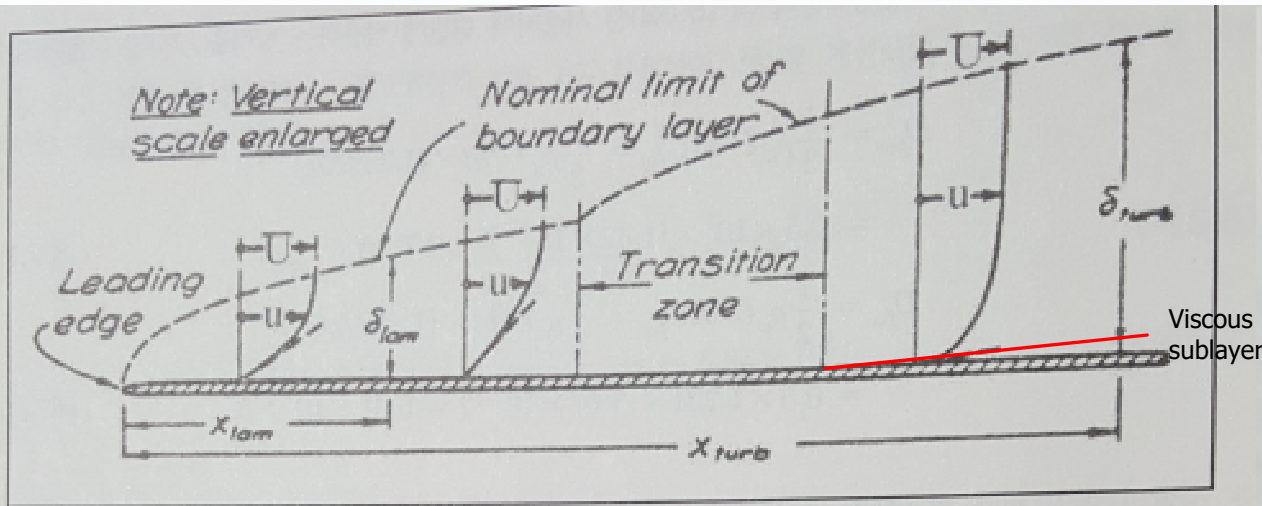


그림 5-10 평행평판을 따라서 형성되는 경계층의 개략도

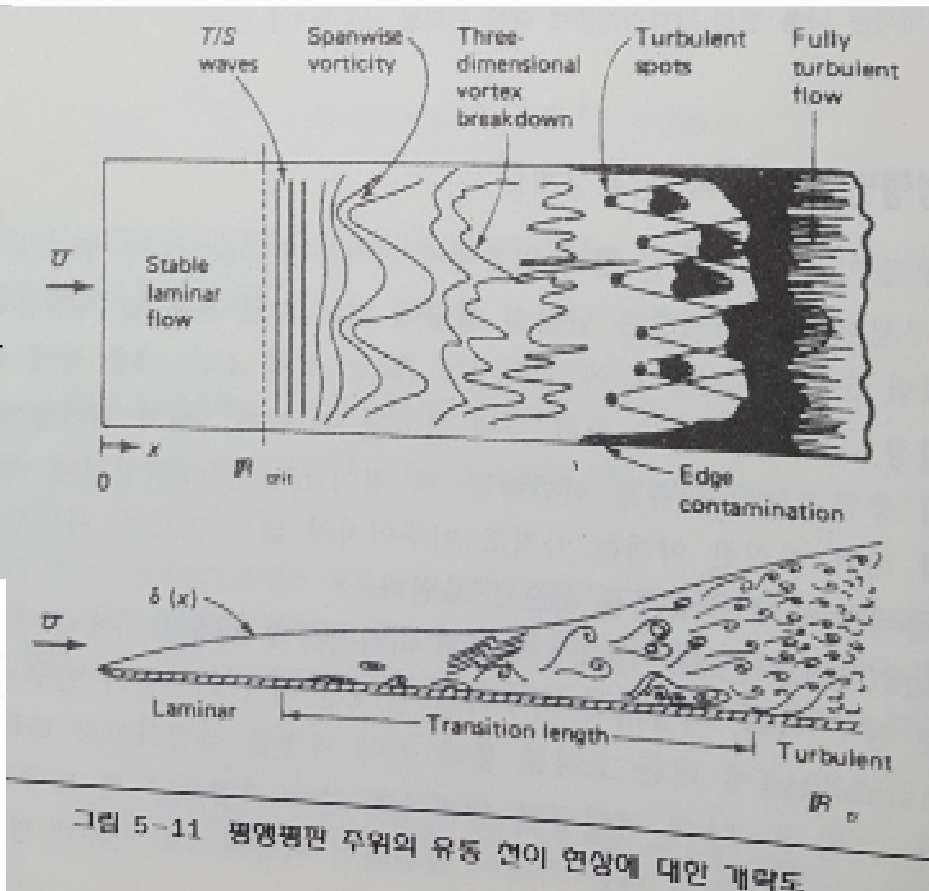
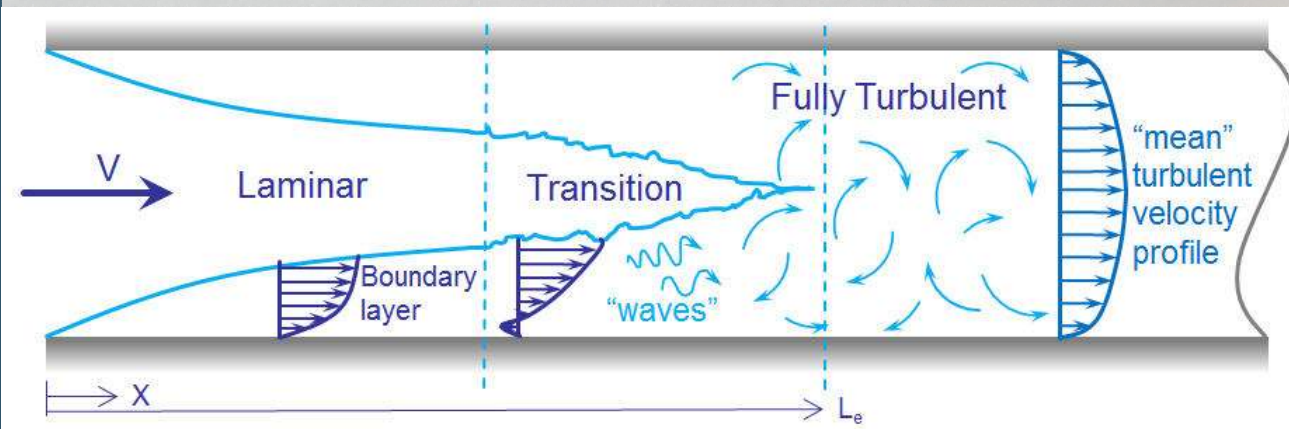
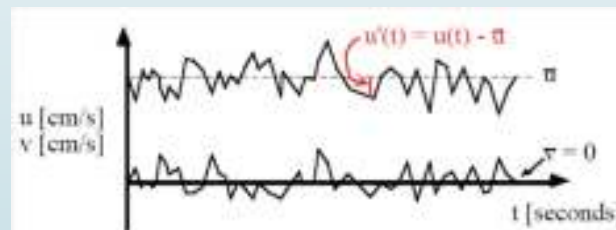
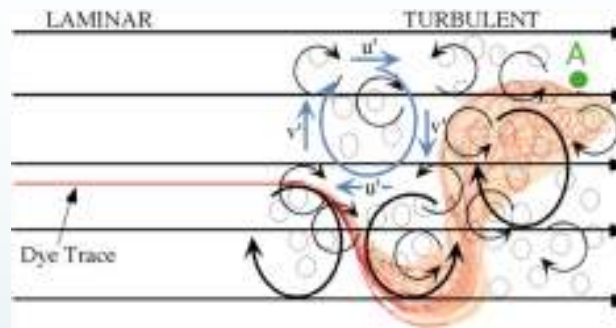


그림 5-11 평행평판 주위의 유동 현상 및 전이 현상에 대한 개략도



평행평판 주위의 난류경계층

- ▶ Prandtl(1933), von Karman(1933)
- ▶ 속도, 압력, 온도가 변수로 작용한다
- ▶ 3차원 유동이다
- ▶ 수많은 eddies 로 구성되어 있다.
- ▶ 섞인다
- ▶ 외부 에너지 공급 없이 수많은 eddies 가 생겨난다.
- ▶ 경계층 전반에 걸쳐 속도 분포가 층류일 때보다 더욱 균일
- ▶ 평판의 마찰항력은 층류일 때보다 증가
- ▶ Reynolds' decomposition
(instantaneous)=(time average)+(fluctuations)
- ▶ Turbulence intensity : ratio of RMS of velocity fluctuations to time-averaged velocity



평행평판 주위의 난류경계층

- Inner layer : 점성 저층, 점성의 영향이 지배적

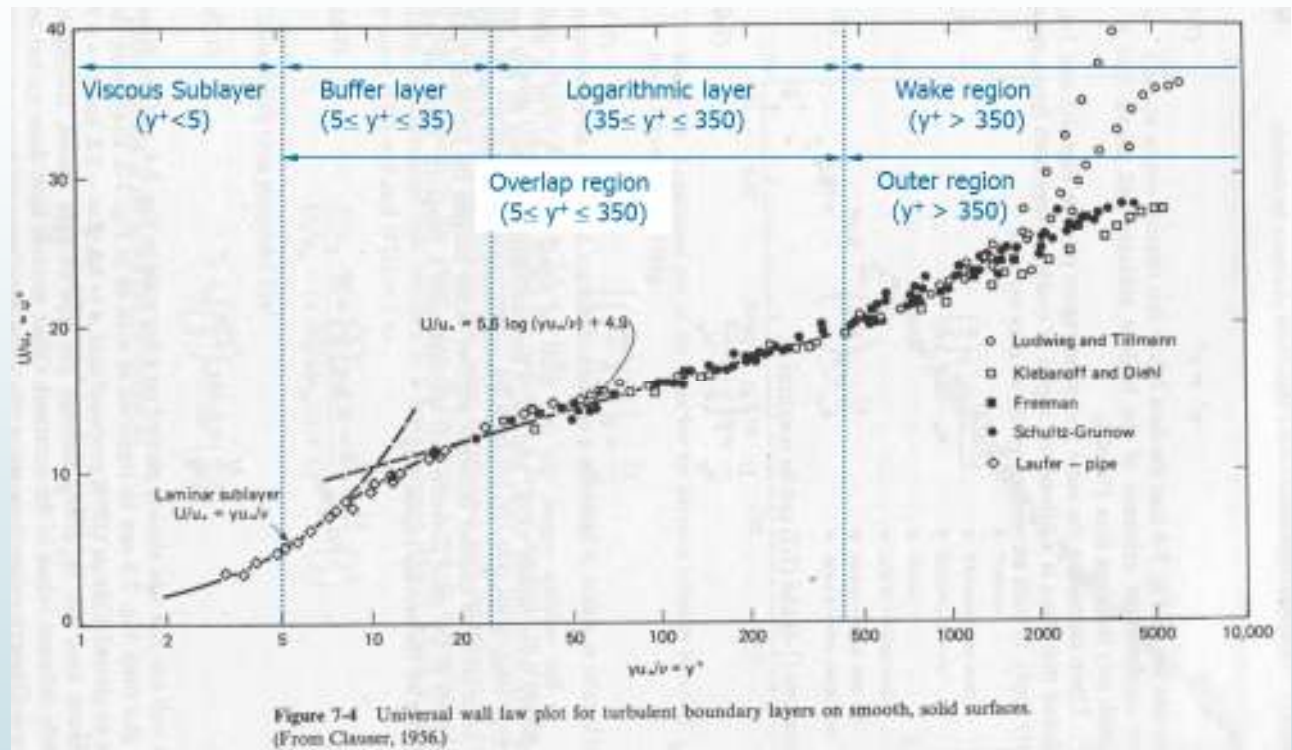
$$\frac{U}{u_*} = \frac{yu_*}{\nu}, \quad u^+ = y^+$$

- Outer layer : 소용돌이의 영향이 지배적

$$(U - U_e)/u_* = f(y/\delta)$$

- Overlap layer : Inner, Outer의 특성이 둘다 나타나는 구역

$$\frac{U}{u_*} = 5.6 \log \left(\frac{yu_*}{\nu} \right) + 5.0$$



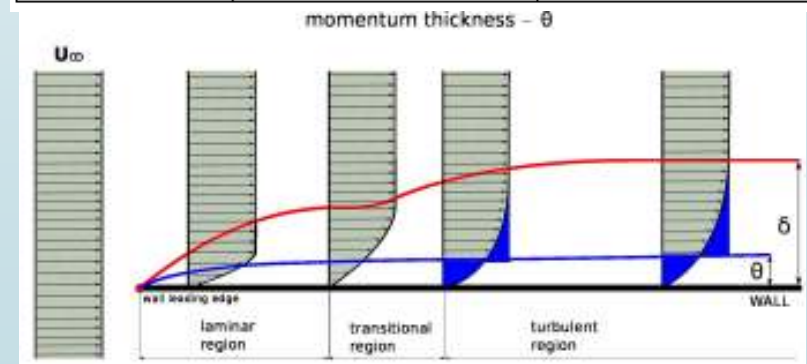
층류 vs 난류

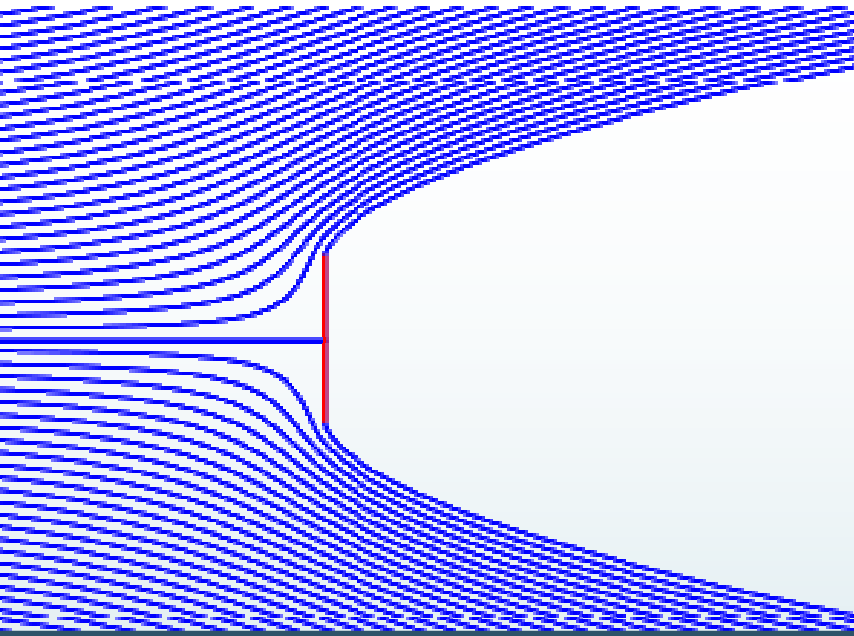
- Boundary layer thickness – 물체 주위의 유동이 점성 영향을 받는 구역의 범위, 유체의 속도가 변하는 구간, 속도가 외부속도의 99%가 되는 범위 ($u=0.99U$)
- Displacement thickness – 점성유체의 경계층 유동에서의 유량과 물체가 원래 위치에서 일정 거리 만큼 떨어진 상태에서 이상유체가 흘렀을때의 유량과 같아 질 수 있는 거리
- Momentum thickness - The momentum thickness, θ , is the distance by which a surface would have to be moved parallel to itself towards the reference plane in an inviscid fluid stream of velocity u_0 to give the same total momentum as exists between the surface and the reference plane in a real fluid
- Local skin friction coefficient – 국부마찰계수(전단응력/동압)
- Wall shear stress – 전단응력
층류 – 속도의 3/2승, 길이의 1/2승에 비례
난류 – 속도의 13/7승, 길이의 1/7승에 비례
같은 조건, 전단응력 난류 > 층류 -> 마찰항력이 크다.
- Drag coefficient – 항력(마찰)계수

[Boundary Layer 1](#)

[Boundary Layer 2](#)

Parameters	Laminar (Blasius solution)	Turbulent (Prandtl approximation)
Boundary layer thickness	$\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}}$	$\frac{\delta}{x} = \frac{0.16}{(Re_x)^{1/7}}$
Displacement thickness	$\frac{\delta^*}{x} = \frac{1.72}{\sqrt{Re_x}}$	$\frac{\delta^*}{x} = \frac{0.02}{(Re_x)^{1/7}}$
Momentum thickness	$\frac{\theta}{x} = \frac{0.664}{\sqrt{Re_x}}$	$\frac{\theta}{x} = \frac{0.016}{(Re_x)^{1/7}}$
Shape factor	$H = \frac{\delta^*}{\theta} = 2.59$	$H = \frac{\delta^*}{\theta} = 1.25$
Local skin friction coefficient	$c_f = \frac{0.664}{\sqrt{Re_x}}$	$c_f = \frac{0.027}{(Re_x)^{1/7}}$
Wall shear stress	$\tau_w = \frac{0.332 \mu^{1/2} \rho^{1/2} U^{3/2}}{x^{1/2}}$	$\tau_w = \frac{0.0135 \mu^{1/7} \rho^{6/7} U^{13/7}}{x^{1/7}}$
Drag coefficient	$c_d = \frac{1.328}{\sqrt{Re_L}}$	$c_d = \frac{0.031}{(Re_L)^{1/7}}$





수직평판, 박리 및 압력 항력

- 항력 = 마찰항력 + 압력항력
- 마찰항력 $\propto$ 점성, 압력항력 $\propto$ 압력차
- Kirchhoff(1869)의 압력항력 가설 - Free streamline

■ 점성 무시

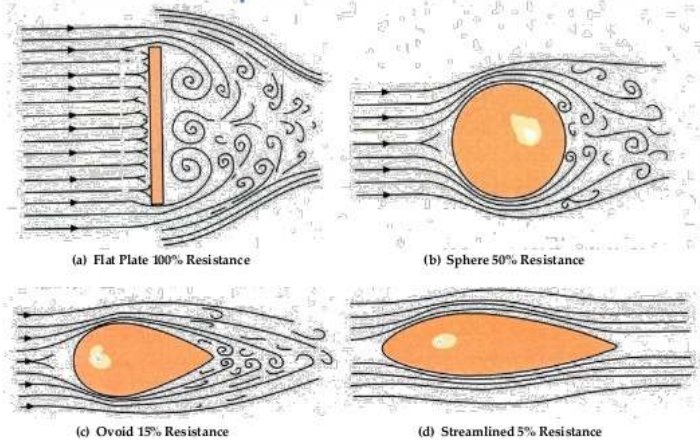
- 수직평판 뒤쪽의 유체는 정지

$$C_d = \frac{D}{A\rho U^2/2} = \frac{2\pi}{\pi + 4} = 0.88$$

- 점성으로 인한 경계층 내부의 회전성유동 -> 경계층의 박리(와동) -> 음압발생

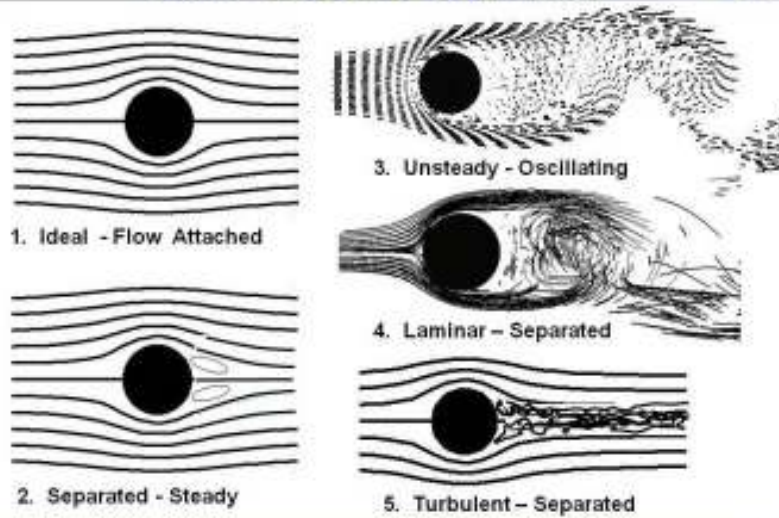
$$C_p = \frac{p}{\rho U^2/2} \cong 2.0$$

Effect of Shapes on Streamlined Flow



STREAMLINED

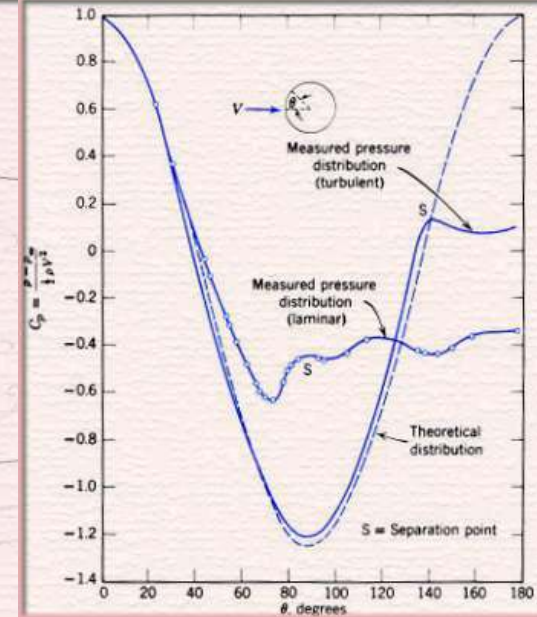
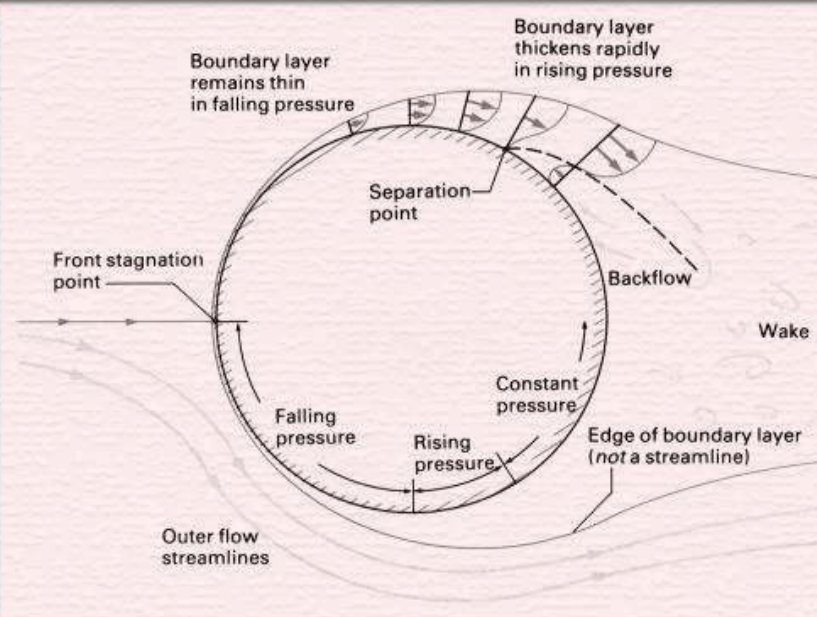
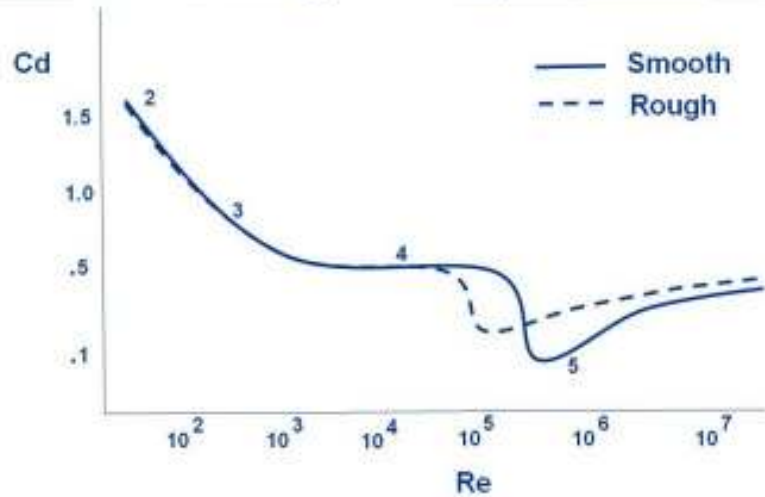
Flow Past a Cylinder



수직평판, 박리 및 압력항력

- ▶ 이상유체일 경우 항력이 발생하지 않는다
- ▶ 박리에 의한 음압 발생 -> 항력
- ▶ 원기둥 주위에 형성된 경계층 내부 유동 종류 -> 난류, 박리점 이동 -> 압력항력 감소

Drag of a Sphere



Accident Troll Field

- Damage to Topside Structure due to Direct Wave Impact (30 Dec 2015)
- Sea state at the time of the accident was less severe than designed
- Rig was considered to vertical wave slam and run up
- Rig design did not comply with minimum 1.5m air gap
- Horizontal wave slam(14m high) crashed into accommodation during mid night. One person died and other workers were evacuated
- Lessoned learn – Horizontal wave slam is important to design in Semisubmersible Floating Unit

